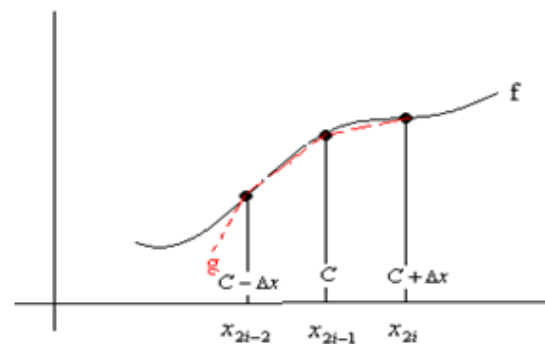


Metode Simpson

Selang $[a, b]$ dibagi menjadi $2n$ bagian sama panjang $\rightarrow \Delta x = \frac{b-a}{2n}$

Pada 3 titik berturutan di kurva f dibuat busur parabola $y = px^2 + qx + r$ $p \neq 0$

Akibatnya $\int_a^b f(x) dx$ dihampiri oleh n buah fungsi kuadrat.



$$f(c - \Delta x) = g_i(c - \Delta x) = p(c - \Delta x)^2 + q(c - \Delta x) + r$$

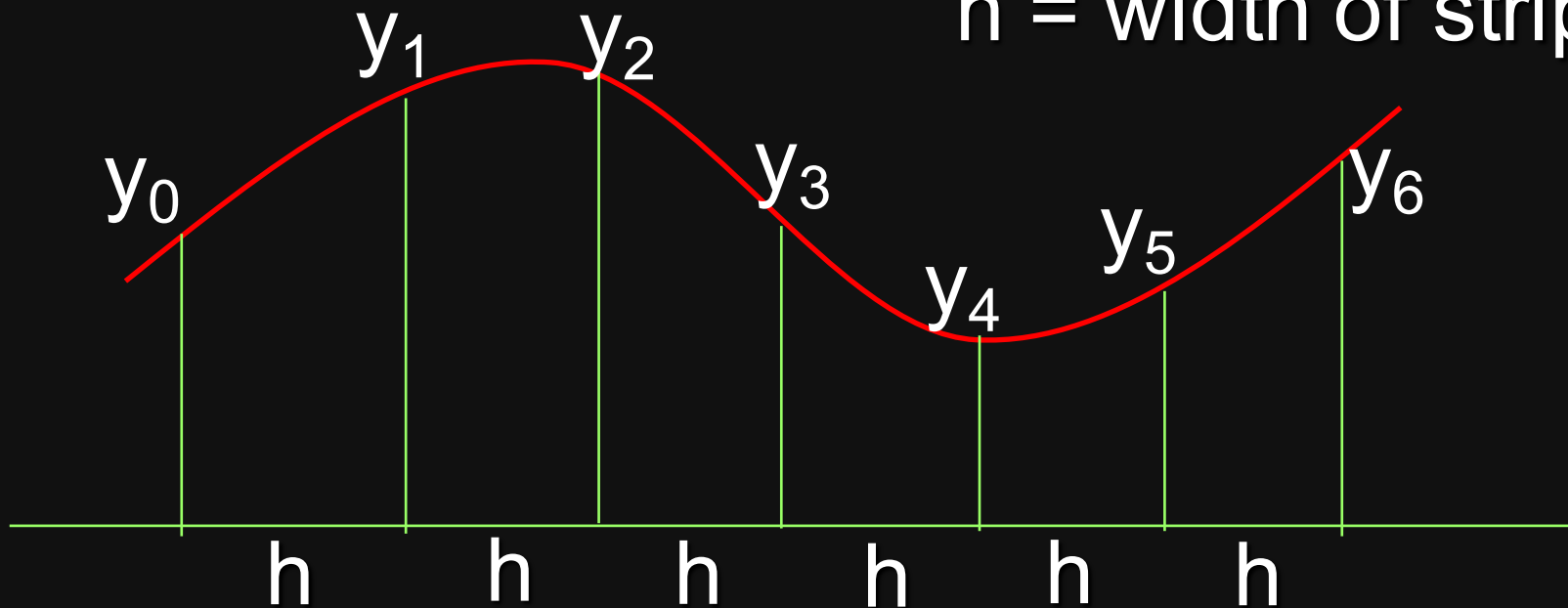
$$\int_{c-\Delta x}^{c+\Delta x} (px^2 + qx + r) dx = \frac{\Delta x}{3} [(6c^2 + 2\Delta x^2)p + 6qc + 6r]$$

SIMPSON'S Rule

The area of the nth strip is estimated to :

$$\frac{1}{3} h (y_{n-1} + 4y_n + y_{n+1})$$

h = width of strip



In this case you get :

$$\text{Area} = \frac{1}{3} h (y_0 + 4y_1 + y_2) + \frac{1}{3} h (y_2 + 4y_3 + y_4) + \frac{1}{3} h (y_4 + 4y_5 + y_6)$$

$$\text{Area} = \frac{1}{3} h (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6)$$

Our example ...

$$\text{Area} = 1/3 h (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6)$$

The end ones.....

You count once

The odd ones.....

You count 4 times

The remaining [even] ones..... You count 2 times

SIMPSON'S Rule

$$\text{area} = \frac{1}{3} h \left(\begin{array}{l} \text{sum of end ordinates} + \\ 4 \times \text{sum of odd ordinates} + \\ 2 \times \text{sum of the remaining even ordinates} \end{array} \right)$$

h = width of strip

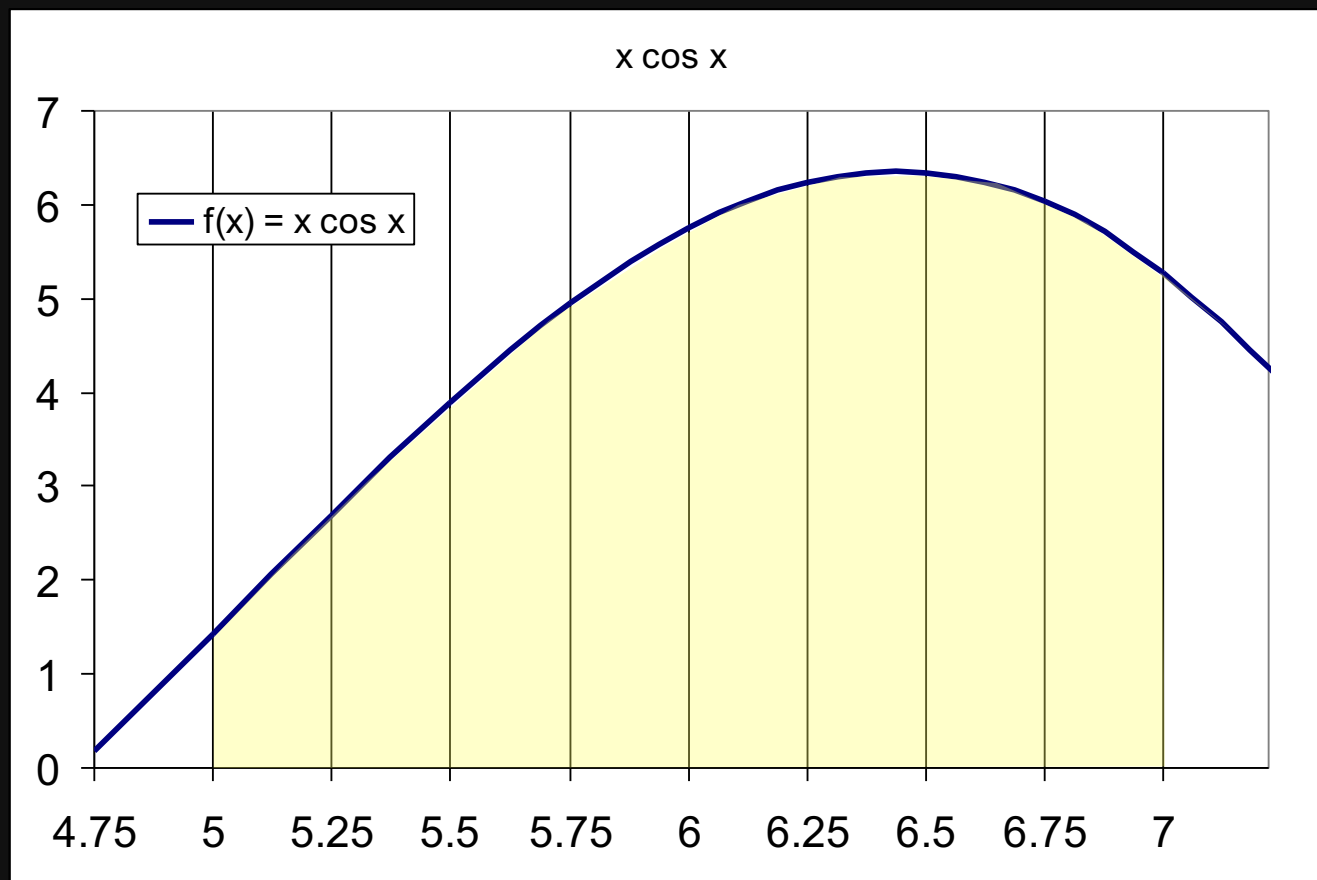
You have to have an even number of strips

SIMPSON'S Rule - Example

$$\int_5^7 x \cos x dx$$

Divide into
8 strips

Width of strips
= $(7-5)/8$
= 0.25



SIMPSON'S Rule - Example

$$\int_5^7 x \cos x dx$$

8 strips

Width (h) = 0.25

ID	0	1	2	3	4	5	6	7	8
x	5	5.25	5.5	5.75	6	6.25	6.5	6.75	7
x cos x	1.418311	2.688449	3.897684	4.951856	5.761022	6.246559	6.34782	6.027793	5.277316



1/3 h	ends	odds	rest
0.0833333	6.695627	19.91466	16.00653

$$area = \frac{1}{3} h \left(\begin{array}{l} \text{sum of end ordinates} + \\ 4 \times \text{sum of odd ordinates} + \\ 2 \times \text{sum of the remaining even ordinates} \end{array} \right)$$

Area = 9.8639 (4 d.p)

Comparison of Methods (1)

Trapezium Rule

Area = $\frac{1}{2} h$ (end ordinates + twice sum of 'interior' ordinates)

Trapezium

$\frac{1}{2} h$
0.125

sum of ends	sum of rest
6.695627	35.92118

Area = 9.8172

Mid-ordinate Rule

Area = width of strip x sum of mid-ordinates

mids	5.125	5.375	5.625	5.875	6.125	6.375	6.625	6.875
x cos x	2.055138258	3.306579	4.449957	5.392326	6.048528	6.348148	6.24173	5.705767

h	sum mids	
0.25	39.54817	= 9.887043

Area = 9.8870

Integration by parts

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx} + c$$

$$\frac{dv}{dx} = \cos x \quad v = \sin x \quad u = x \quad \frac{du}{dx} = 1$$

Substitute in

$$\int x \cos x = x \sin x - \int \sin x dx$$

Leads to

$$\int x \cos x = x \sin x + \cos x$$

$$\int_5^7 x \cos x = \left[x \sin x + \cos x \right]_5^7$$

$$= 7 \sin 7 + \cos 7 - 5 \sin 5 - \cos 5 = 9.8637676$$

Comparison of Methods (2)

	Area	Error	%
Algebra	9.8637676	0	0.0000
Simpson's	9.863942	-0.00017441	-0.0018
Trapezium	9.8172488	0.046518789	0.4738
Mid-ordinate	9.8870433	-0.02327571	-0.2354

Numerical methods all based on 8 strips.

Errors will change depending on
the number of strips used.

Simpson's Rule provides, by far,
the most accurate estimate

Simpson's Rule Practice

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Core 3

exercise D

... in case you thought I'd forgotten the obvious picture

